7.4 The Inverse Laplace Transform

Sug. HW: odds 1–29

Question What function $f(t)$ has Laplace transform $\frac{1}{(s-2)^2}$?

Table (#. $k=2, n=1$) gives

$$\mathcal{L}\{e^{2t}t\} = \frac{1!}{(s-2)^2} = \frac{1}{(s-2)^2}.$$

So $f(t) = e^{2t}t$ has Laplace transform $\frac{1}{(s-2)^2}$.

$$\mathcal{L}\{e^{2t}t\} = \frac{1}{(s-2)^2}.$$

We write $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\} = te^{2t}$.

It can be shown that there is no other continuous function whose Laplace transform is te^{2t}.

Question What function $f(t)$ has Laplace transform $\frac{-s}{(s^2+4)^2}$?

$$\mathcal{L}\{t\sin 2t\} = \frac{4s}{(s^2+4)^2} \quad \text{for } s>0 \text{ (Table (14))}.$$

$$\mathcal{L}\left\{-\frac{1}{4}t\sin 2t\right\} = -\frac{1}{4}\left(\frac{4s}{(s^2+4)^2}\right) = \frac{-s}{(s^2+4)^2}.$$

So $\mathcal{L}^{-1}\left\{\frac{-s}{(s^2+4)^2}\right\} = -\frac{t}{4}\sin 2t.$