Definition Given a function $F(s)$, if there is a function $f(t)$ that is continuous on $[0,\infty)$ and satisfies

$$\mathcal{L}\{f\} = F(s)$$

then we say $f(t)$ is the inverse Laplace transform of $F(s)$ and write: $f = \mathcal{L}^{-1}\{F\}$.

Linearity of inverse transform

Suppose $\mathcal{L}^{-1}\{F\}$, $\mathcal{L}^{-1}\{f_1\}$, $\mathcal{L}^{-1}\{f_2\}$ exist and continuous on $[0,\infty)$. Let c, c_1, c_2 be any constants. Then

(1) $\mathcal{L}^{-1}\{f_1 + f_2\} = \mathcal{L}^{-1}\{f_1\} + \mathcal{L}^{-1}\{f_2\}$

(2) $\mathcal{L}^{-1}\{cf\} = c\,\mathcal{L}^{-1}\{f\}$

(3) $\mathcal{L}^{-1}\{c_1 f_1 + c_2 f_2\} = c_1\mathcal{L}^{-1}\{f_1\} + c_2\mathcal{L}^{-1}\{f_2\}$.

Example (#6) Find $\mathcal{L}^{-1}\left\{\frac{3}{(2s+5)^3}\right\}$.

We use table #4: $\mathcal{L}\{e^{kt}t^n\} = \frac{n!}{(s-k)^{n+1}}$.

$$\frac{3}{(2s+5)^3} = \frac{3}{[2(s+5/2)]^3} = \frac{3}{8(s+5/2)^3}$$

Let $k = -5/2$, $n = 2$.

$$\mathcal{L}\{e^{-5t/2}t^2\} = \frac{2!}{(s+5/2)^3}$$

$$\mathcal{L}^{-1}\left\{\frac{3}{8(s+5/2)^3}\right\} = \mathcal{L}^{-1}\left\{\frac{3}{16}\,\frac{2!}{(s+5/2)^3}\right\}$$

$$= 3/16\,\mathcal{L}^{-1}\left\{\frac{2!}{(s+5/2)^3}\right\} = 3/16\, e^{-5t/2}t^2.$$