

(14)

Let $R(s) = \frac{P(s)}{Q(s)}$ be a rational function of s where $\deg P < \deg Q$. $Q(s)$ we can compute $\mathcal{L}^{-1}\{R(s)\}$ using the method of partial fractions.

Case 1 $Q(s)$ has only non repeated linear factors.

Example Find $\mathcal{L}^{-1}\left\{\frac{s+11}{(s-1)(s+3)}\right\}$.

$$\frac{s+11}{(s-1)(s+3)} = \frac{A}{s-1} + \frac{B}{s+3}$$

$$s+11 = A(s+3) + B(s-1)$$

$$s=1: \quad 12 = 4A, \quad A=3.$$

$$s=-3 \quad 8 = -4B, \quad B=-2.$$

$$\frac{s+11}{(s-1)(s+3)} = \frac{3}{s-1} - \frac{2}{s+3}$$

$$\begin{aligned} \mathcal{L}^{-1}\left\{\frac{s+11}{(s-1)(s+3)}\right\} &= 3\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} \\ &= 3e^t - 2e^{-3t}. \end{aligned}$$

In general, if $Q(s) = (s-r_1)(s-r_2)\cdots(s-r_n)$ where $r_1, r_2, \dots, r_n$ are all different then

$$\frac{P(s)}{Q(s)} = \frac{A_1}{s-r_1} + \frac{A_2}{s-r_2} + \cdots + \frac{A_n}{s-r_n}$$

for some constant $A_1, A_2, \dots, A_n$ when $\deg P < \deg Q$.