Case 2 Repeated Linear Factors

Example Find $\mathcal{L}^{-1}\left\{ \dfrac{4s^2-11s+12}{(s-2)^2(s+1)} \right\}$.

$$\frac{4s^2-11s+12}{(s-2)^2(s+1)} = \frac{A}{s-2} + \frac{B}{(s-2)^2} + \frac{C}{s+1}$$

$$4s^2-11s+12 = A(s-2)(s+1) + B(s+1) + C(s-2)^2.$$

$s=2 \Rightarrow \quad 16-22+12 = 3B, \quad 3B=6, \quad B=2.$

$s=-1 \Rightarrow \quad 4+11+12 = 9C, \quad 9C=27, \quad C=3.$

$s=0 \Rightarrow \quad 12 = -2A + B + 4C = -2A + 2 + 12,$

$-2A = -2, \quad A=1.$

So

$$\frac{4s^2-11s+12}{(s-2)^2(s+1)} = \frac{1}{s-2} + \frac{2}{(s-2)^2} + \frac{3}{(s+1)}$$

$$\mathcal{L}^{-1}\left\{ \frac{4s^2-11s+12}{(s-2)^2(s+1)} \right\} = e^{2t} + 2te^{2t} + 3e^{-t}.$$

Note: In general, a factor $(s-r)^m$ in $Q(s)$ gives rise to terms

$$\frac{A_1}{(s-r)} + \frac{A_2}{(s-r)^2} + \cdots + \frac{A_m}{(s-r)^m}$$

in the partial fraction expansion of $\frac{P(s)}{Q(s)}$.