

(16)

Case 3 Quadratic Factors

Example Find $\int^{-1} \left\{ \frac{(5x-4)(x-5)}{(x^2-4x+12)(x+1)} \right\}$

$$\frac{(5x-4)(x-5)}{(x^2-4x+12)(x+1)} = \frac{Ax+B}{x^2-4x+12} + \frac{C}{x+1}$$

since $x^2-4x+12$ has discr. $\Delta = b^2-4ac$
 $= 16 - 4 \cdot 12 = 16 - 48 < 0.$

So

$$(5x-4)(x-5) = (Ax+B)(x+1) + C(x^2-4x+12).$$

$$x=-1 \Rightarrow (-9)(-6) = C(1+4+12)$$

$$54 = 18C, \quad C=3.$$

$$(5x-4)(x-5) - 3(x^2-4x+12) = (Ax+B)(x+1)$$

$$5x^2-29x+20 - 3x^2+12x-36 = (Ax+B)(x+1)$$

$$2x^2-17x-16 = (Ax+B)(x+1) = Ax^2 + \dots$$

$$A=2 \quad (\text{coeff of } x^2).$$

$$x=0 \Rightarrow -16 = B.$$

Hence

$$\frac{(5x-4)(x-5)}{(x^2-4x+12)(x+1)} = \frac{2x-16}{x^2-4x+12} + \frac{3}{x+1}$$

$$x^2-4x+12 = (x-2)^2 + 9 = (x-2)^2 + 3^2.$$

$$\frac{2x-16}{x^2-4x+12} = \frac{2x-16}{(x-2)^2+3^2} = \frac{2(x-2)-12}{(x-2)^2+3^2}$$

$$= \frac{2(x-2)}{(x-2)^2+3^2} - \frac{12}{(x-2)^2+3^2}$$