

7.5 Solving Initial Value Problems

(15)

Suggested HW:

Given an IVP

$$ay'' + by' + cy = g(t), \quad y(0) = y_0, \quad y'(0) = y_1.$$

We let $Y(s) = \mathcal{L}\{y(t)\}$.

$$\text{Then } \mathcal{L}\{y'(t)\} = sY - y(0) = sY - y_0$$

$$\mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0) = s^2Y - sy_0 - y_1.$$

Example (#4) Solve the IVP

$$y'' + 6y' + 5y = 12e^t, \quad y(0) = -1, \quad y'(0) = 7.$$

Let $Y = \mathcal{L}\{y\}$. Then

$$\mathcal{L}\{y'\} = sY - y(0) = sY + 1$$

$$\mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0) = s^2Y + s - 7.$$

$$\mathcal{L}\{y'' + 6y' + 5y\} = \mathcal{L}\{12e^t\}$$

$$s^2Y + s - 7 + 6(sY + 1) + 5Y = \frac{12}{s-1}$$

$$s^2Y + s - 7 + 6(sY + 1) + 5Y = \frac{12}{s-1}$$

$$(s^2 + 6s + 5)Y + s - 1 = \frac{12}{s-1}$$

$$(s^2 + 6s + 5)Y = \frac{12}{s-1} - (s-1)$$

$$(s+1)(s+5)Y = \frac{12 - (s-1)^2}{(s-1)}$$

$$Y = \frac{12 - (s-1)^2}{(s+1)(s+5)(s-1)}$$