$$\frac{12-(s-1)^2}{(s+1)(s+5)(s-1)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+5}$$

$$12-(s-1)^2 = A(s+1)(s+5) + B(s-1)(s+5) + C(s-1)(s+1)$$

$s=1 \Rightarrow \quad 12 = 12A, \quad A=1.$

$s=-1 \Rightarrow \quad 8 = -8B, \quad B=-1.$

$s=-5 \Rightarrow \quad 12-36 = C(-6)(-4)$

$-24 = 24C, \quad C=-1.$

So

$$Y = \frac{1}{s-1} - \frac{1}{s+1} - \frac{1}{s+5}.$$

$$y = \mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\}$$

$$= e^{t} - e^{-t} - e^{-5t}.$$

Example Solve the IVP

$$y'' + 6y = t^2 - 1, \quad y(0)=0, \quad y'(0)=-1$$

for $Y = \mathcal{L}\{y\}$.

Let $Y = \mathcal{L}\{y\}$.

$$\mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0) = s^2Y + 1$$

$$\mathcal{L}\{t^2-1\} = \frac{2}{s^3} - \frac{1}{s}.$$

$$\mathcal{L}\{y'' + 6y\} = \mathcal{L}\{t^2-1\}$$

$$s^2Y + 1 + 6Y = \frac{2}{s^3} - \frac{1}{s}$$