

$$Y(s^2+6) = \frac{2}{s^3} - \frac{1}{s} - 1$$

$$Y(s) = \frac{1}{s^2+6} \left[\frac{2}{s^3} - \frac{1}{s} - 1 \right], \text{ for } s > 0.$$

#36 $ty'' - ty' + y = 2, \quad y(0)=2, \quad y'(0)=-1.$

Let $Y = \mathcal{L}\{y\}.$

$$\mathcal{L}\{y'\} = sY - y(0) = sY - 2.$$

$$\mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0) = s^2Y - 2s + 1$$

$$\mathcal{L}\{ty'\} = -\frac{d}{ds} \mathcal{L}\{y'\} = -\frac{d}{ds} (sY - 2)$$

$$= -(sY' + Y)$$

$$\mathcal{L}\{ty''\} = -\frac{d}{ds} \mathcal{L}\{y''\} = -\frac{d}{ds} (s^2Y - 2s + 1)$$

$$= -(s^2Y' + 2sY - 2)$$

$$\mathcal{L}\{ty'' - ty' + y\} = \mathcal{L}\{2\}$$

$$-(s^2Y' + 2sY - 2) + (sY' + Y) + Y = \frac{2}{s}$$

$$(-s^2 + s)Y' + (2 - 2s)Y = -2 + \frac{2}{s}$$

$$Y' + \frac{2(1-s)}{s(-s+1)} Y = \frac{-2s+2}{s(s-s^2)}$$

$$Y' + \frac{2}{s} Y = \frac{2(1-s)}{s^2(1-s)} = \frac{2}{s^2}.$$