Let $a \geq 0$.

$$\mathcal{L}\{H(t-a)\} = \mathcal{L}\{u(t-a)\}$$

$$= \int_0^\infty e^{-st} H(t-a)\, dt$$

$$= \int_0^a e^{-st} H(t-a)\, dt + \int_a^\infty e^{-st} H(t-a)\, dt$$

$$= \int_a^\infty e^{-st}\, dt$$

$$\int_a^N e^{-st} dt = \left.\frac{e^{-st}}{-s}\right]_a^N = \frac{e^{-sa}}{s} - \frac{e^{-sN}}{s}$$

$$\lim_{N\to\infty} \int_a^N e^{-st}\, dt = \lim_{N\to\infty}\left(\frac{e^{-sa}}{s} - \frac{e^{-sN}}{s}\right)$$

$$= \frac{e^{-sa}}{s} \quad \text{if } s > 0.$$

$$\boxed{\mathcal{L}\{H(t-a)\} = \mathcal{L}\{u(t-a)\} = \frac{e^{-sa}}{s} \quad \text{for } s > 0}$$

<u>Theorem</u> Let $a > 0$. Suppose

$$F(s) = \mathcal{L}\{f\} \quad \text{for } s > \alpha \geq 0$$

Then

$$\mathcal{L}\{f(t-a)\, u(t-a)\} = e^{-as} F(s).$$

<u>Proof</u>

$$\mathcal{L}\{f(t-a)\, u(t-a)\} = \int_0^\infty f(t-a)\, u(t-a)\, e^{-st}\, dt$$

$$= \int_a^\infty f(t-a)\, e^{-st}\, dt \qquad (\text{Let } v = t-a,\ dv = dt)$$

$$= \int_0^\infty f(v)\, e^{-s(v+a)}\, dv$$