Express $g(t)$ in terms of unit step functions & find $\mathcal{L}\{g(t)\}$.

t	0 to 1	1 to 2	2 to 3	after 3
$u(t-3)$	0	0	0	1
$u(t-2)-u(t-3)$	0	0	1	0
$(t-2)(u(t-2)-u(t-3))$	0	0	$t-2$	0

$$g(t) = (t-2)\,(u(t-2)-u(t-3)) + u(t-3)$$
$$= (t-2)\,u(t-2) - (t-3)\,u(t-3)$$
$$\mathcal{L}\{g(t)\} = \mathcal{L}\{(t-2)\,u(t-2)\} - \mathcal{L}\{(t-3)\,u(t-3)\}$$
$$= e^{-2s}\,\mathcal{L}\{t\} - e^{-3s}\,\mathcal{L}\{t\}$$
$$= \frac{1}{s^2}\left(e^{-2s} - e^{-3s}\right) \qquad (\text{for } s>0).$$

<u>Example</u> (#14) Determine $\mathcal{L}^{-1}\left\{\dfrac{e^{-3s}}{s^2+9}\right\}$.

$$\mathcal{L}\{\tfrac{1}{3}\sin 3t\} = \frac{1}{s^2+9}.$$

$$\mathcal{L}\{u(t-3)\,\tfrac{1}{3}\sin 3(t-3)\} = e^{-3s}\,\mathcal{L}\{\tfrac{1}{3}\sin 3t\} = \frac{e^{-3s}}{s^2+9}.$$

Hence,

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2+9}\right\} = \frac{1}{3}\,u(t-3)\,\sin 3(t-3).$$