<u>Example</u> (# 18) Determine $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}(3s^2-s+2)}{(s-1)(s^2+1)}\right\}$

$$\frac{3s^2-s+2}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+1} \quad \text{(partial fracs)}$$

$$3s^2-s+2 = A(s^2+1) + (Bs+C)(s-1).$$

$s=1 \Rightarrow \quad 4 = 2A, \; A=2.$

Coeff of s^2: $\quad 3 = A+B = 2+B, \; B=1.$

$s=0 \Rightarrow \quad 2 = A-C = 2-C, \; C=0.$

So

$$\frac{3s^2-s+2}{(s-1)(s^2+1)} = \frac{2}{s-1} + \frac{s}{s^2+1}$$

$$\mathcal{L}^{-1}\left\{\frac{3s^2-s+2}{(s-1)(s^2+1)}\right\} = 2e^{t} + \cos t = f(t).$$

$$\mathcal{L}\{u(t-1)f(t-1)\} = e^{-s}\mathcal{L}\{f(t)\} = e^{-s}\frac{(3s^2-s+2)}{(s-1)(s^2+1)}.$$

Hence, $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}(3s^2-s+2)}{(s-1)(s^2+1)}\right\} = u(t-1)f(t-1)$

$$= u(t-1)\left(2e^{t-1} + \cos(t-1)\right).$$