

#30  $y'' + y = u(t-2) - u(t-4)$ ,  $y(0)=1, y'(0)=0$

now:  $u(t-2)$   $u(t-4)$   $u(t-2) - u(t-4)$   $\left| \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right. \left| \begin{array}{c} 2 \\ 0 \\ 1 \end{array} \right. \left| \begin{array}{c} 4 \\ 1 \\ 0 \end{array} \right.$

$g(t) = u(t-2) - u(t-4) = \begin{cases} 1 & \text{if } 2 < t < 4 \\ 0 & \text{otherwise} \end{cases}$

Let  $Y = \mathcal{L}\{y\}$

$\mathcal{L}\{y''\} = s^2 Y - s y(0) - y'(0) = s^2 Y - s$   
 $\mathcal{L}\{u(t-2) - u(t-4)\} = \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}$

$s^2 Y - s + Y = \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}$

$(s^2 + 1) Y = s + \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}$

$Y = \frac{s}{s^2 + 1} + \frac{1}{(s^2 + 1)s} (e^{-2s} - e^{-4s})$

$\frac{1}{(s^2 + 1)s} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1}$

$1 = A(s^2 + 1) + (Bs + C)s$

$s=0 \quad A=1$

Coef of  $s^2$ :  $0 = A + B, \quad B = -1$

Coef of  $s$ :  $0 = C$

$\frac{1}{s(s^2 + 1)} = \frac{1}{s} + \frac{-s}{s^2 + 1}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = 1 - \cos t = f(t)$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s(s^2+1)}$$

$$\mathcal{L}\{u(t-2)f(t-2)\} = e^{-2s}\mathcal{L}\{f\} = \frac{e^{-2s}}{s(s^2+1)}$$

$$\mathcal{L}\{u(t-4)f(t-4)\} = e^{-4s}\mathcal{L}\{f\} = \frac{e^{-4s}}{s(s^2+1)}$$

Therefore

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s^2+1)}\right\} - \mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s(s^2+1)}\right\}$$

$$= \cos t + (1 - \cos(t-2))u(t-2) - (1 - \cos(t-4))u(t-4)$$

$$= \begin{cases} \cos t & 0 \leq t < 2 \\ 1 + \cos t - \cos(t-2) & 2 < t < 4 \\ \cos t + \cos(t-4) - \cos(t-2) & t > 4 \end{cases}$$

# PLOTTING SOLUTION USING MAPLE

(29)

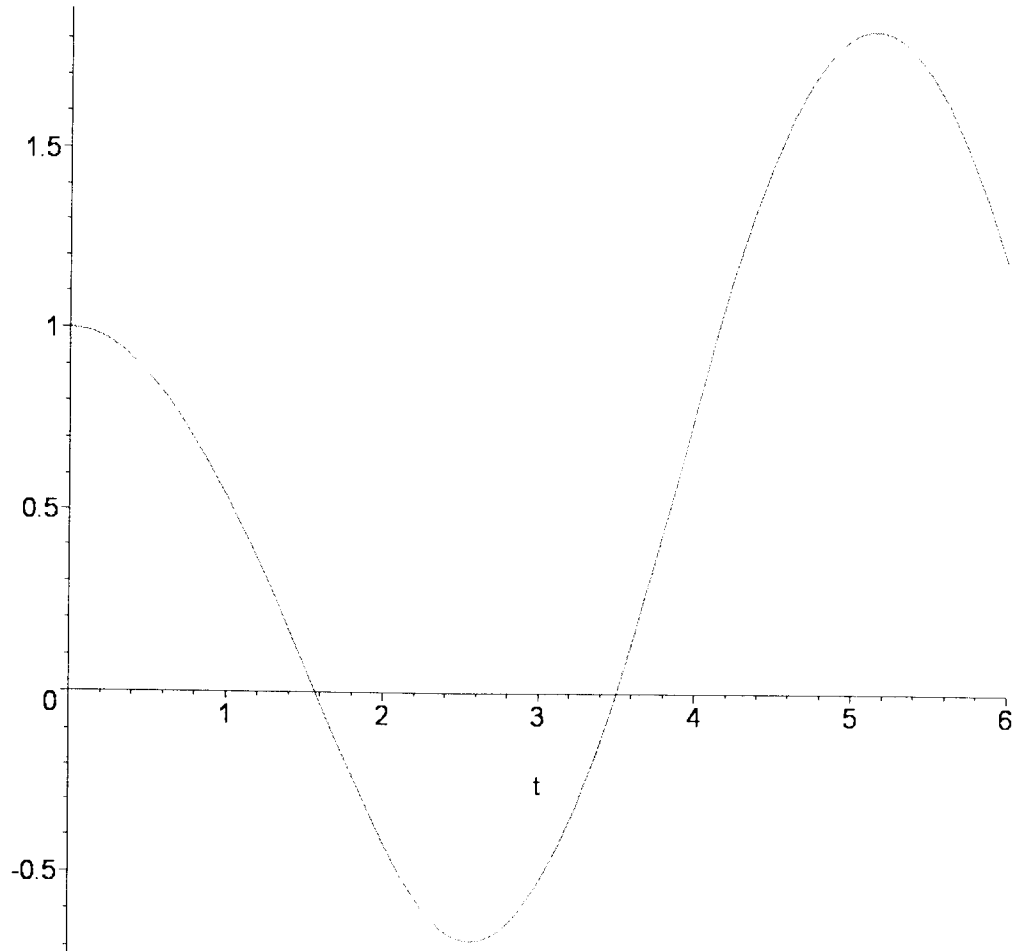
```
> dsolve({diff(y(t), t, t) + y(t) = Heaviside(t-2) - Heaviside(t-4), D(y)(0) = 0, y(0) = 1}, y(t));
```

$$y(t) = \cos(t) + (1 - \cos(t-2)) \text{Heaviside}(t-2) + \text{Heaviside}(t-4) (-1 + \cos(t-4))$$

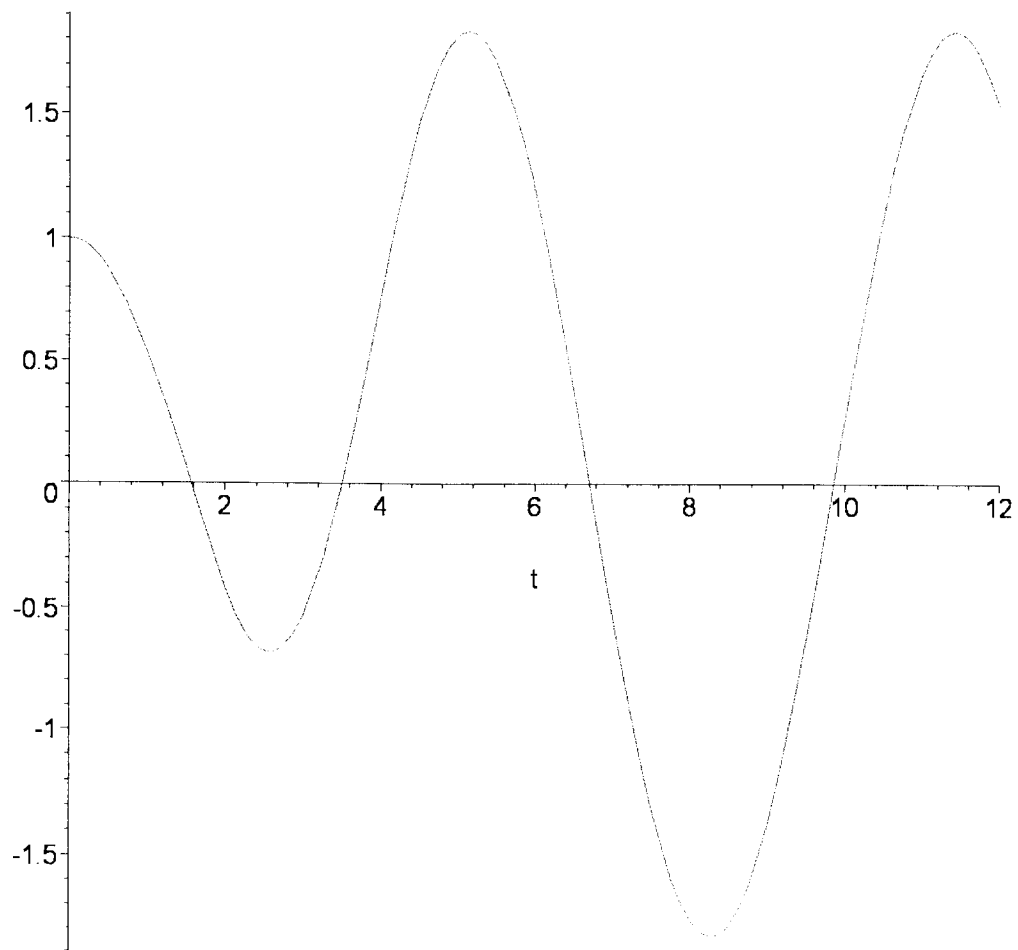
```
> Y:=rhs(%);
```

$$Y := \cos(t) + (1 - \cos(t-2)) \text{Heaviside}(t-2) + \text{Heaviside}(t-4) (-1 + \cos(t-4))$$

```
> plot(Y, t=0..6);
```



```
> plot(Y, t=0..12);
```



>