

7.8 Impulses & the Dirac Delta Function

Log HW odds 1-29.

The Dirac delta function $\delta(t)$ has the property that

$$\int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0)$$

$$\delta(t) = \begin{cases} 0 & \text{if } t \neq 0 \\ \infty & \text{if } t = 0. \end{cases}$$

Problem (See Eg. p. 111 of text).

A mass is attached to a spring is released from rest 1/m below equilibrium for the spring mass system

$$x'' + 9x = 0.$$

After π seconds the mass is struck by a hammer exerting an impulse of 3 units on the mass. Determine $x(t)$.

$$x'' + 9x = 0, \quad 0 \leq t \leq \pi, \quad x(0) = 1, \quad x'(0) = 0.$$

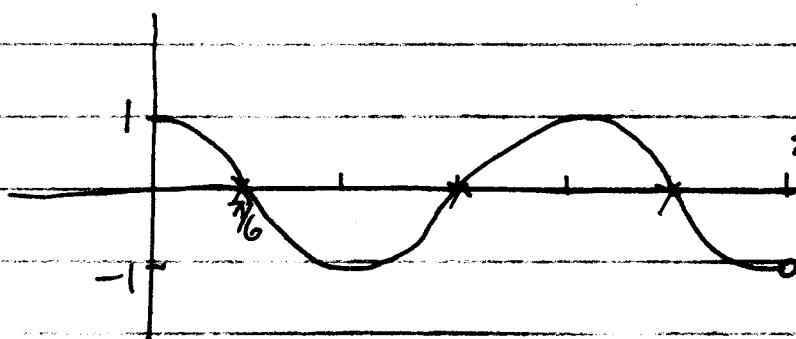
$$x = c_1 \cos 3t + c_2 \sin 3t \quad (\text{A.E.: } r^2 + 9 = 0, \quad r = \pm 3i).$$

$$x(0) = c_1 = 1$$

$$x' = -3c_1 \sin 3t + 3c_2 \cos 3t,$$

$$x'(0) = 3c_2 = 0 \quad \& \quad c_2 = 0.$$

$$\text{So } x = \cos 3t, \quad 0 \leq t < \pi.$$



note

$$x(\pi^-) = -1$$

$$x'(\pi^-) = 0$$