The impulse causes a sudden change in momentum (mv) at $t=\pi$.

We solve $\quad x'' + 9x = 0, \quad x(\pi) = -1, \; x'(\pi) = 3, \quad t \geq \pi.$

Two independent solns are $\; x_1 = \cos 3t, \; x_2 = \sin 3t.$

$$x = c_1 \cos 3t + c_2 \sin 3t$$

$$x(\pi) = c_1 \cos 3\pi = c_1 \cos \pi = -c_1 = -1, \quad c_1 = 1.$$

$$x' = -3c_1 \sin 3t + 3c_2 \cos 3t$$

$$x'(\pi) = 3c_2 \cos 3\pi = -3c_2 = 3, \quad c_2 = -1.$$

So $\quad x(t) = \cos 3t - \sin 3t$

Hence,

$$x(t) = \begin{cases} \cos 3t & 0 \leq t < \pi \\ \cos 3t - \sin 3t & t \geq \pi. \end{cases}$$

$$\mathcal{L}\{\delta(t)\} = \int_0^\infty e^{-st}\,\delta(t)\,dt = 1.$$

Let $t_0 > 0$.

$$\mathcal{L}\{\delta(t-t_0)\} = \int_0^\infty e^{-st}\,\delta(t-t_0)\,dt$$

$$= \int_{-\infty}^\infty e^{-st}\,\delta(t-t_0)\,dt = \int_{-\infty}^\infty e^{-s(u+t_0)}\,\delta(u)\,du \qquad \begin{pmatrix} t-t_0 = u \\ dt = du \end{pmatrix}$$

$$= e^{-st_0}.$$

__NOTE__ : $F(t) = M\,\delta(t-t_0)$ represents an impulse of M units at $t=t_0$.