CHAPTER 8 Series Solutions of Differential Equations

Let $p(x) = a + bx + cx^2 + dx^3$.

$p(0) = a$

$p'(x) = b + 2cx + 3dx^2$

$p'(0) = b$

$p''(x) = 2c + 3\cdot 2\cdot d x$

$p''(0) = 2c \qquad c = \dfrac{p''(0)}{2}$

$p'''(x) = 3\cdot 2\cdot d$

$p'''(0) = 3\cdot 2\, d \qquad d = \dfrac{p'''(0)}{3\cdot 2}$.

If $p(x)$ is a cubic polynomial then

$$p(x) = p(0) + p'(0)x + \frac{p''(0)}{2}x^2 + \frac{p'''(0)}{(3)(2)}x^3.$$

If $p(x)$ is a polynomial of degree n then

$$p(x) = p(0) + p'(0)\,x + \frac{p''(0)\,x^2}{2!} + \frac{p'''(0)\,x^3}{3!} + \dots + \frac{p^{(n)}(0)\,x^n}{n!}$$

$$p(x) = p(x_0) + p'(x_0)(x-x_0) + \frac{p''(x_0)(x-x_0)^2}{2!} + \dots + \frac{p^{(n)}(x_0)(x-x_0)^n}{n!}$$

Definition The nth degree Taylor polynomial of $f(x)$ near $x = x_0$ (or centered at $x = x_0$) is

$$p_n(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \dots + \frac{f^{(n)}(x_0)(x-x_0)^n}{n!}$$

$$= \sum_{j=0}^{n} \frac{f^{(j)}(x_0)}{j!}(x-x_0)^j.$$