<u>Example</u> $f(x) = e^x \qquad x_0 = 0.$

$f(0)=1,\ f'(0)=1,\ f''(0)=1,\ f^{(n)}(0)=1$ for all n.

The Taylor polynomials of e^x near $x_0=0$ are

$$p_0(x) = 1$$

$$p_1(x) = 1 + x$$

$$p_2(x) = 1 + x + \frac{x^2}{2!}$$

$$p_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}.$$

The <u>Taylor Series</u> of $f(x)$ near $x = x_0$ is

$$\sum_{j=0}^{\infty} \frac{f^{(j)}(x_0)}{j!}(x-x_0)^j = f(x_0) + f'(x_0)(x-x_0) + \cdots$$

Under certain conditions this converges to $f(x)$.

<u>Eg</u> $\quad e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

converges to e^x for all x.

<u>Example</u> (# 3) First the first three nonzero terms in The Taylor polynomial approximation of the IVP

$$y'(x) = \sin y + e^x \qquad y(0) = 0.$$

$$y'(0) = \sin y(0) + e^0 = \sin 0 + 1 = 1.$$

$$y'' = (\cos y)\, y' + e^x$$

$$y''(0) = \cos(y(0))\, y'(0) + 1 = (\cos 0)\, 1 + 1 = 2$$

$$y''' = (\cos y)\, y'' - \sin y\, (y')^2 + e^x$$

$$y'''(0) = (1)(2) - 0 + 1 = 3$$

<u>Ans</u> $\quad y(0) + y'(0)\frac{x^1}{1!} + y''(0)\frac{x^2}{2!} + \cdots = x + \frac{2x^2}{2} + \frac{3x^3}{3!} + \cdots$