## 8.2 Power Series & Analytic Functions

(3) $a_n \searrow 0$ $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

SERIES (1) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ conv for $p > 1$. (2) $\sum a_n$ conv $\Rightarrow \lim_{n\to\infty} a_n = 0$.

A power series near $x_0$ (or about $x_0$) is an infinite series

$$(*) \qquad \sum_{n=0}^{\infty} a_n (x - x_0)^n = a_0 + a_1(x - x_0) + \cdots$$

where $x$ is a variable & $a_n$ are constants.

$(*)$ converges absolutely if $\sum_{n=0}^{\infty} |a_n (x - x_0)^n|$ converges.

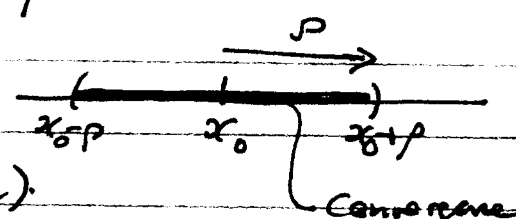
Note If a series converges absolutely then it converges.

### Radius of Convergence Theorem

Given a power series $(*)$. There is a real number $\rho \geq 0$

$0 \leq \rho \leq \infty$ (radius of convergence) such that

$(*)$ converges absolutely for $|x - x_0| < \rho$ & diverges for $|x - x_0| > \rho$




$\rho = 0$ Only converges for $x = x_0$.

$\rho = \infty$ (converges absolutely for all $x$).

**Example** The series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots$

converges absolutely for $|x| < 1$ & diverges for $|x| > 1$.

In fact $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $-1 < x < 1$.

The series diverges for $|x| \geq 1$.