RATIO TEST

Let $\quad r = \lim_{n\to\infty} \left| \frac{b_{n+1}}{b_n} \right|.$

If $r < 1$ Then $\sum_{n=1}^{\infty} b_n$ converges absolutely.

If $r > 1$ Then $\sum_{n=1}^{\infty} b_n$ diverges.

RATIO TEST

Let $\quad L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|.$

Then the radius of convergence of the series $\sum_{n=0}^{\infty} a_n(x-x_0)^n$ is

(1) $\rho = 1/L$ if $L > 0$

(2) $\rho = \infty$ if $L = 0$

(3) $\rho = 0$ if $L = \infty$

Example (#1) Determine the convergence of $\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-1)^n$.

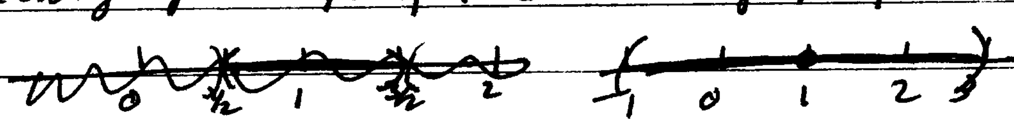
Let $\quad a_n = \frac{2^{-n}}{n+1}.$

$$\frac{a_{n+1}}{a_n} = \frac{2^{-(n+1)}}{n+2} \cdot \frac{n+1}{2^{-n}} = \left(\frac{n+1}{n+2}\right)\left(\frac{1}{2}\right)$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{1}{2}\left(\frac{n+1}{n+2}\right)$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(\frac{1+1/n}{1+2/n}\right) = \frac{1}{2}\cdot\frac{1}{1} = \frac{1}{2}.$$

So series converges for $|x-1| < 2$ & div. for $|x-1| > 2$



$x = 3\cancel{/2}$ $\quad \sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-1)^n = \sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges

$x = \cancel{1/2}-1$ $\quad \sum_{n=0}^{\infty} (-1)^n/_{n+1}$ converges by Alt. Series test since $1/_{n+1} \searrow 0$.