Hence convergence set $= [-1, 3)$.

<u>Theorem</u> Suppose
$$f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$
has a positive radius of convergence ρ.
Then

(1) f is d'ble for $|x - x_0| < \rho$ &
$$f'(x) = \sum_{n=1}^{\infty} n a_n (x - x_0)^{n-1} \quad \text{for } |x - x_0| < \rho$$
& series cvges for $|x - x_0| < \rho$.

(2)
$$\int f(x)\,dx = \sum_{n=0}^{\infty} a_n \frac{(x - x_0)^{n+1}}{n+1} + C \quad \text{for } |x - x_0| < \rho.$$

<u>Cor 1.</u> Suppose $f(x) = \sum a_n (x - x_0)^n$ has positive radius of convergence ρ. Then
$$a_n = \frac{f^{(n)}(x_0)}{n!}.$$

<u>Cor 2</u> Suppose $\sum_{n=0}^{\infty} a_n (x - x_0)^n = 0$ for all x in an open interval. Then $a_n = 0$ for all $n \geq 0$.

<u>Example</u> (#12) Find first three nonzero terms in power series expansion of $(\sin x)(\cos x)$.

$$\sin x = x - \frac{x^3}{3!} - \frac{x^5}{5!} + \cdots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sin x \cos x = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)$$

$$= x + \left(-\tfrac{1}{2} - \tfrac{1}{6}\right) x^3 + \left(\frac{1}{4!} + \frac{1}{3!\,2!} + \frac{1}{5!}\right) x^5 + \cdots$$

$$= x - \tfrac{2}{3} x^3 + \tfrac{2}{15} x^5 + \cdots$$