<u>Theorem</u>

Suppose $f(x) = \sum_{n=0}^{\infty} a_n (x-x_0)^n$, $g(x) = \sum_{n=0}^{\infty} b_n (x-x_0)^n$ converge for $|x-x_0|<\rho$. Then

$$f(x)\,g(x) = \sum_{n=0}^{\infty} c_n (x-x_0)^n$$ converges for $|x-x_0|<\rho$ &

$$c_n = \sum_{k=0}^{n} a_k\, b_{n-k}.$$ (Cauchy products).

<u>Definition</u> A function $f(x)$ is <u>analytic</u> at $x_0$ if ~~$f(x) = \sum_{n=0}^{\infty} a_n (x-x_0)^n$~~ some power series ~~___~~ near $x_0$ with +ve radius of convergence converges to $f(x)$.

$$f(x) = \sum_{n=0}^{\infty} a_n (x-x_0)^n \quad \text{for } |x-x_0|<\rho.$$

<u>Note</u> If $f$ is analytic ~~___~~ at $x = x_0$ then

$$a_n = \frac{f^{(n)}(x_0)}{n!} \quad \text{for } n \geq 0.$$