We want two linearly independent solns. (10)

For $\boxed{y_1}$ we want $y_1(0)=1$, $y_1'(0)=0$.

$$a_0 = 1$$
$$a_1 = 0$$
$$a_2 = \frac{2a_0}{2} = 1$$
$$a_3 = \frac{2a_1}{3} = 0$$
$$a_4 = \frac{2a_2}{4} = 1/2$$

Hence

$$y_1(x) = 1 + x^2 + \tfrac{1}{2}x^4 + \cdots$$

For $\boxed{y_2}$ $y_2(0) = 0$, $y_2'(0) = 1$.

Now

$$a_0 = 0$$
$$a_1 = 1$$
$$a_2 = \frac{2a_0}{2} = 0$$
$$a_3 = \frac{2a_1}{3} = 2/3$$
$$a_4 = \frac{2a_2}{4} = 0$$
$$a_5 = \frac{2a_3}{5} = \frac{2^2}{15} = \frac{4}{15}$$

Hence $y_2(x) = x + 2/3\, x^3 + 4/15\, x^5 + \cdots$

Note $y_1(x), y_2(x)$ are linearly indept. since

$$W[y_1,y_2](0) = \begin{vmatrix} y_1(0) & y_2(0) \\ y_1'(0) & y_2'(0) \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0.$$