$$a_{2n} = \frac{1}{n!} a_0$$

$$a_{2n+1} = \frac{2 \cdot 2 \cdot 2 \cdot \cdots \cdot 2}{(2n+1)(2n-1) \cdots\cdots 3} a_1 = \frac{2^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)} a_1$$

Hence

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} a_{2n} x^{2n} + \sum_{n=0}^{\infty} a_{2n+1} x^{2n+1}$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} a_0 + \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdot 5 \cdots (2n+1)} x^{2n+1} a_1$$

$$= a_0 y_1 + a_1 y_2 \qquad (a_0, a_1 \text{ any constants})$$

&

$$y_1 = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} = e^{x^2}$$

$$y_2 = \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdot 5 \cdots (2n+1)} x^{2n+1}.$$