8.4 Equations with analytic coefficients

Theorem Suppose x_0 is an ordinary point of the DE

(*) $y'' + p(x)y' + q(x)y = 0$.

Then (*) has two linearly independent solutions that are analytic at x_0. Moreover the radius of convergence of ~~the general soln~~ any soln

$$y = \sum_{n=0}^{\infty} a_n (x-x_0)^n$$

is at least the distance of x_0 to the nearest singular point (real or complex).

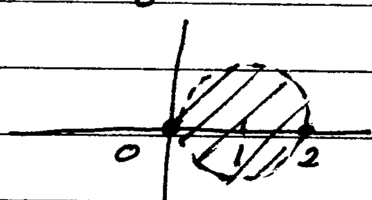
Example (#9)

$$(x^2-2x)y'' + 2y = 0, \qquad x_0 = 1.$$

$$y'' + \frac{2}{x^2-2x} y = 0.$$

$p(x) = 0 \qquad q(x) = \frac{2}{x(x-2)}$ is not analytic at $x=0, 2$.

Any soln near $x_0 = 1$ has radius of convergence $\rho \geq 1$.



$$y = \sum_{n=0}^{\infty} a_n (x-1)^n$$

$$y' = \sum_{n=0}^{\infty} a_n (x-1)^{n-1} n$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n (x-1)^{n-2}$$