$$x^2 - 2x = (x-1)^2 - 1$$

$$(x^2-2x)y'' = \sum_{n=0}^{\infty} n(n-1)a_n(x-1)^n - \sum_{n=0}^{\infty} n(n-1)a_n(x-1)^{n-2}$$

$$= \sum_{n=0}^{\infty} \left[n(n-1)a_n - (n+2)(n+1)a_{n+2} \right](x-1)^n$$

$$(x^2-2x)y'' + 2y = \sum_{n=0}^{\infty} \left[(n^2-n+2)a_n - (n+2)(n+1)a_{n+2} \right](x-1)^n$$

$$(n+2)(n+1)a_{n+2} = (n^2-n+2)a_n \qquad n \geq 0$$

$$a_{n+2} = \frac{(n^2-n+2)\,a_n}{(n+2)(n+1)}$$

$$a_2 = a_0$$

$$a_3 = \frac{2}{3 \cdot 2} a_1 = \frac{1}{3} a_1$$

$$a_4 = \frac{(4-2+2)}{(4)(3)} a_2 = \frac{1}{3} a_2 = \frac{1}{3} a_0$$

Hence

$$y = a_0 + a_1(x-1) + a_0(x-1)^2 + \tfrac{1}{3}(x-1)^3 a_1 + \tfrac{1}{3}(x-1)^4 a_0 + \cdots$$

$$= a_0\left(1 + (x-1)^2 + \cdots\right)$$

$$+ a_1\left((x-1) + \tfrac{1}{3}(x-1)^3 + \cdots\right)$$

for any constants a_0, a_1 & $|x-1| < 1$.