#15

$(x^2+1)y'' - e^x y' + y = 0 \qquad y(0)=1,\ y'(0)=1.$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$(x^2+1)y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} n(n-1) a_n x^n$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$-e^x y' = \left(-1 - x - \frac{x^2}{2} - \frac{x^3}{6} + \cdots\right)(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots)$$

$$= -a_1 + (-2a_2 - a_1)x + \left(-\frac{a_1}{2} - 2a_2 - 3a_3\right)x^2 + \cdots$$

$$(x^2+1)y'' = (x^2+1)(2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \cdots)$$

$$= 2a_2 + 6a_3 x + (2a_2 + 12a_4)x^2 + \cdots$$

x^0: $a_0 - a_1 + 2a_2 = 0$

$2a_2 = 0 \qquad a_2 = 0$

x^1: $a_1 - a_1 - 2a_2 + 6a_3 = 0$

$a_3 = 0$

x^2: $a_2 - \frac{a_1}{2} - 2a_2 - 3a_3 + 2a_2 + 12a_4 = 0$

$-\frac{1}{2} + 12a_4 = 0 \qquad a_4 = \frac{1}{24}$

x^3: $a_3 - \frac{1}{6} - 4a_4 + (20a_5) = 0$

$20a_5 = \frac{1}{6} + 4a_4 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$a_5 = \frac{1}{60}$