

(16)

$$y = 1 + x + \frac{1}{24}x^4 + \frac{1}{60}x^5 + \dots$$

$$\#25 \quad (1+x^2)y'' - xy' + y = e^{-x}$$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

$$y' = a_1 + 2a_2x + 3a_3x^2 + \dots$$

$$-xy' = -a_1x - 2a_2x^2 - 3a_3x^3 + \dots$$

$$y'' = 2a_2 + 6a_3x + \dots$$

$$x^2y'' = 2a_2x^2 + \dots$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots$$

$$(1+x^2)y'' - xy' + y$$

$$= (a_0 + 2a_2) + (a_1 + 6a_3)x + \dots$$

$$= 1 - x + \dots$$

$$2a_2 = 1 - a_0$$

$$6a_3 = -1$$

$$a_3 = -\frac{1}{6}$$

$$y = a_0 + a_1x + \left(\frac{1}{2} - \frac{a_0}{2}\right)x^2 - \frac{1}{6}x^3 + \dots,$$

where a_0, a_1 are any constants.