

(19)

Example

#3 $x^2 y'' - xy' + 17y = 0 \quad (x > 0).$

$$y = x^r$$

A.E: $r(r-1) - r + 17 = 0$

$$r^2 - 2r + 17 = 0$$

$$(r-1)^2 + 16 = 0$$

$$(r-1)^2 = -16$$

$$r-1 = \pm 4i$$

$$r = 1 \pm 4i$$

General soln is

~~general soln~~

$$x^r = e^{(\ln x)(1+4i)} = e^{(\ln x)} e^{4i \ln x}$$

$$= x (\cos(4 \ln x) + i \sin(4 \ln x)).$$

General soln is given by

$$y = c_1 x \cos(4 \ln x) + c_2 x \sin(4 \ln x)$$

where c_1, c_2 are constants.