8.6 Method of Frobenius

~~we consider DE~~

~~$y'' + p(x)y' + q(x)y = 0$~~

~~where~~

Defn x_0 is a regular singular point of the DE

$$y'' + p(x)y' + q(x)y = 0$$

if $(x - x_0)p(x)$ and $(x-x_0)^2 q(x)$ are analytic at x_0. A singular point is called irregular if it is not regular.

NOTE $x = 0$ is a regular singular pt. of the Cauchy-Euler eqn.

$$y'' + \frac{b}{ax}y' + \frac{c}{ax^2}y = 0$$

Since $x\,p(x) = b/a$

& $x^2 q(x) = c/a$ are constant & hence analytic.

Example Classify the singular points of Legendre's equation

$$(1-x^2)y'' - 2xy' + (\alpha)(\alpha+1)y = 0$$

where α is a constant.

$$p(x) = \frac{-2x}{1-x^2} = \frac{-2x}{(1-x)(1+x)}$$

$$q(x) = \frac{\alpha(\alpha+1)}{1-x^2} = \frac{\alpha(\alpha+1)}{(1-x)(1+x)}$$

are ~~any~~ analytic except for $x = \pm 1$.