The singular points are $x = \pm 1$.

$\boxed{x=1}$ $(x-1)\,p(x) = \dfrac{-2x\,(x-1)}{(1-x)(1+x)} = \dfrac{2x}{1+x}$ is analytic at $x=1$.

$(x-1)^2 q(x) = \dfrac{\alpha(\alpha+1)(x-1)^2}{(1-x)(1+x)} = \dfrac{-\alpha(\alpha+1)(x-1)}{1+x}$ is analytic at $x=1$.

So $x=1$ is a regular singular point.

$\boxed{x=-1}$ $(x+1)\,p(x) = \dfrac{2x}{1-x}$ is analytic at $x=-1$.

$(x+1)^2 q(x) = \dfrac{-\alpha(\alpha+1)(x+1)}{(1-x)}$ is analytic at $x=-1$.

So $x=-1$ is a regular singular point.

Discussion assume $x=0$ is a regular singular point of $y'' + p(x)y' + q(x)y = 0$.

Then

$x\,p(x) = p_0 + p_1 x + p_2 x^2 + \cdots$ is analytic at $x=0$

$x^2 q(x) = q_0 + q_1 x + \cdots$ is analytic at $x=0$.

Suppose

$$y = x^r(a_0 + a_1 x + \cdots) = a_0 x^r + a_1 x^{r+1} + \cdots$$

is a solution with $a_0 \neq 0$.

$$q(x)\,y = x^r(a_0 + a_1 x + \cdots)\frac{1}{x^2}(q_0 + q_1 x + \cdots$$

$$= x^{r-2}(a_0 q_0 + \cdots)$$