$$p(x)\, y' = \frac{1}{x}(p_0 + p_1 x + \cdots)\left[x^r(a_1 + \cdots) + r x^{r-1}(a_0 + \cdots)\right]$$

$$= \frac{1}{x}(p_0 + p_1 x + \cdots)\, x^{r-1}\left[r a_0 + \cdots \right]$$

$$= x^{r-2}(r a_0 p_0 + \cdots)$$

$$y'' = \left[a_0 r(r-1) x^{r-2} + \cdots \right]$$

$$q(x)\, y'' = $$

Hence coeff of x^{r-2} in

$$y'' + p(x) y' + q(x) y$$

is

$$a_0 r(r-1) + r a_0 p_0 + a_0 q_0$$

$$= a_0 (r(r-1) + r p_0 + q_0) = 0$$

We require

$$r(r-1) + r p_0 + q_0 = 0 \quad \text{(Indicial equation)}$$

Note $p_0 = \lim_{x\to 0} x\, p(x)$ & $q_0 = \lim_{x \to 0} x^2 q(x)$.

~~Definition~~

Definition Let x_0 be a regular singular point of

$$y'' + p(x) y' + q(x) y = 0.$$

The <u>indicial equation</u> of x_0 is

$$r(r-1) + r p_0 + q_0 = 0$$

where $p_0 = \lim_{x \to x_0} (x - x_0)\, p(x)$ & $q_0 = \lim_{x \to x_0} (x - x_0)^2 q(x)$

Here

$$p(x) = \frac{p_0}{x - x_0} + p_1 + p_2(x - x_0) + \cdots$$

$$q(x) = \frac{q_0}{(x - x_0)^2} + \frac{q_1}{(x - x_0)} + \cdots$$

The roots of the indicial equation are called <u>exponents</u>.