

Example Find the indicial eqn & exponents
of the DE

$$x^2 y'' + x y' + (x^2 - \frac{1}{4}) y = 0 \quad (x > 0).$$

(Bessel's eqn mod k)

$$\Leftrightarrow y'' + \frac{1}{x} y' + \frac{(x^2 - \frac{1}{4})}{x^2} y = 0$$

$$x p(x) = x - \frac{1}{x} = 1 \quad p_0 = 1$$

$$x^2 q(x) = (x^2 - \frac{1}{4}) = -\frac{1}{4} + x^2 \quad q_0 = -\frac{1}{4}$$

Indicial eqn:

$$r(r-1) + p_0 r + q_0 = 0$$

$$r(r-1) + r - \frac{1}{4} = 0 \quad (\text{Indicial Eqn})$$

$$r^2 - \frac{1}{4} = 0$$

$$r = \pm \frac{1}{2} \quad (\text{Exponents}).$$

Example (#19)

$$9x^2 y'' + 9x^2 y' + 2y = 0 \quad (x > 0).$$

Find the first few nonzero terms in the series exp. about $x=0$ for a soln.

$$p(x) = 1 \quad q(x) = \frac{2}{9x^2}$$

$$x p(x) = x \quad \text{so } p_0 = 0$$

$$x^2 q(x) = \frac{2}{9} \quad \text{so } q_0 = \frac{2}{9}$$

Indicial Eqn:

$$r(r-1) + p_0 r + q_0 = 0$$

$$r(r-1) + \frac{2}{9} = 0$$

$$r^2 - r + \frac{2}{9} = 0$$

$$r^2 - r + \frac{1}{4} = \frac{1}{4} - \frac{2}{9} = \frac{9-8}{36} = \frac{1}{36}$$