

$$(r - \frac{1}{2})^2 = \frac{1}{6} \quad (24)$$

$$r - \frac{1}{2} = \pm \frac{1}{6}$$

$$r = \frac{1}{2} \pm \frac{1}{6} \Rightarrow \frac{2}{3}, \frac{1}{3}$$

Take $r = \frac{2}{3}$.

$$\text{Let } y = x^{2/3} (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots)$$

$$y = a_0 x^{2/3} + a_1 x^{5/3} + a_2 x^{8/3} + a_3 x^{11/3} + \dots$$

$$y' = \frac{2}{3} a_0 x^{-1/3} + \frac{5}{3} a_1 x^{2/3} + \frac{8}{3} a_2 x^{5/3} + \frac{11}{3} a_3 x^{8/3} + \dots$$

$$x^2 y' = 6 a_0 x^{5/3} + 15 a_1 x^{8/3} + \frac{24}{3} a_2 x^{11/3} + \dots$$

$$y'' = -\frac{2}{9} a_0 x^{-4/3} + \frac{10}{9} a_1 x^{-1/3} + \frac{40}{9} a_2 x^{2/3} + \frac{88}{9} a_3 x^{5/3} + \dots$$

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$$2y = 2 a_0 x^{2/3} + 2 a_1 x^{5/3} + 2 a_2 x^{8/3} + 2 a_3 x^{11/3} + \dots$$

$$x^2 y'' + x^2 y' + 2y = 0$$

$$12 a_1 + 6 a_0 = 0$$

$$12 a_1 = -6 a_0$$

$$a_1 = -\frac{1}{2} a_0$$

$$42 a_2 + 15 a_1 = 0$$

$$14 a_2 + 5 a_1 = 0$$

$$14 a_2 = -5 a_1 = \frac{5}{2} a_0$$

$$a_2 = \frac{5}{28} a_0$$

$$90 a_3 + \frac{24}{3} a_2 = 0$$

$$15 a_3 + 8 a_2 = 0$$

$$a_3 = -\frac{8}{15} a_2 = -\frac{8}{15} \cdot \frac{5}{28} a_0 = -\frac{1}{3 \cdot 7} a_0 = -\frac{1}{21} a_0$$