$$2(1-x)y' = \sum_{n=0}^{\infty} 2(n+r)\, a_n\, x^{n+r-1} - \sum_{n=0}^{\infty} 2(n+r) a_n x^{n+r}$$

$$= 2r\, a_0\, x^{r-1} + \sum_{n=1}^{\infty} 2(n+r) a_n x^{n+r-1} - \sum_{n=0}^{\infty} 2(n+r) a_n x^{n+r}$$

$$= 2r\, a_0\, x^{r-1} + \sum_{n=0}^{\infty} 2(n+r+1)\, a_{n+1}\, x^{n+r} - \sum_{n=0}^{\infty} 2(n+r) a_n x^{n+r}$$

$$3xy'' = \sum_{n=0}^{\infty} 3(n+r)(n+r-1)\, a_n x^{n+r-1}$$

$$= 3(r)(r-1)\, a_0 x^{r-1} + \sum_{n=1}^{\infty} 3(n+r)(n+r-1)\, a_n x^{n+r-1}$$

$$= 3r(r-1)\, a_0 x^{r-1} + \sum_{n=0}^{\infty} 3(n+r+1)(n+r)\, a_{n+1}\, x^{n+r}$$

x^{r-1}: $\quad a_0\,(3r(r-1) + 2r) = 0$

x^{n+r}: $\quad 3(n+r+1)(n+r)\, a_{n+1} + 2(n+r+1)\, a_{n+1} - 2(n+r)\, a_n \cancel{= 0}$

$$\cancel{(n+r+1)(3n+3r+2)\, a_{n+1} = 2(n+r)\, a_n}$$

$$\cancel{a_{n+1} = \frac{2(n+r)\, a_n}{(n+r+1)(3n+3r+2)} \quad \text{for } n \geq 0}$$

$-4\, a_n$

$$(n+r+1)(3n+3r+2)\, a_{n+1} - 2(n+r+2)\, a_n = 0$$

$(y(x))$ $$a_{n+1} = \frac{2(n+r+2)\, a_n}{(n+r+1)(3n+3r+2)} \quad \text{for } n \geq 0$$

Note $(n+r+1)(3n+3r+2) \neq 0$ since $n \geq 0$ & $r = 0, \frac{1}{3}$.