Is there a soln that is decreasing for $x > 0$? (27)

$$y = \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} a_n (n+r) x^{n-1}$$

$n + r \geq 0$ for $n \geq 0$.

We want $a_n < 0$. Just take $a_0 = -1$.

Then by (*) $a_n < 0$ for all $n \geq 0$.

Note:

(1) For y_1 let $r = 1/3$.

$$a_{n+1} = \frac{2(n + 7/3)\, a_n}{(n + 4/3)(3n+3)} \quad \text{for } n \geq 0.$$

$(3n+4)(n+1)$

Let $a_0 = 1$.

$$a_1 = \frac{2 \cdot 7/3}{4} = 7/6$$

$$a_2 = \frac{2 \cdot 10/3}{7 \cdot 2} a_1 = \frac{10}{3} \cdot \frac{1}{7} \cdot \frac{7}{6} = \frac{5}{9}$$

$$y_1 = x^{\frac{1}{3}}\left(1 + 7/6\, x + 5/9\, x^2 + \cdots\right)$$

(2) For y_2 let $r = 0$.

$$a_{n+1} = \frac{2(n+2)\, a_n}{(n+1)(3n+2)}$$

$a_0 = 1$

$$a_1 = \frac{2 \cdot 2}{2} a_0 = 2$$

$$a_2 = \frac{2 \cdot 3}{2 \cdot 5} a_1 = 3/5 \cdot 2 = 6/5$$

$$y_2 = x^0 \left(1 + 2x + 6/5\, x^2 + \cdots\right)$$