LINEAR DIFFERENTIAL EQUATIONS (Ch 4 & 6)

Linear Transformations Let A, B be sets we write $L: A \to B$ if $L(a) \in B$ for each $a \in A$.

Example ① Let $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$.

Note $f(x) = x^2 \in \mathbb{R}$ if $x \in \mathbb{R}$.

② Let $L: \mathbb{R}^2 \to \mathbb{R}^2$ where

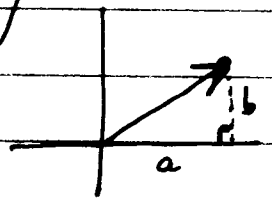
$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ x_1 - x_2 \end{pmatrix}$$

Note: (i) $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$ if $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$.

(ii) $\mathbb{R}^2 = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : a, b \in \mathbb{R} \right\}$

This is set of vectors in $\mathbb{R}^2$.

$$\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$$



A transformation $L: A \to B$ is **linear** if

$$L(a_1 + a_2) = L(a_1) + L(a_2)$$

and $L(ca) = cL(a)$

for $a, a_1, a_2 \in A$, $c \in \mathbb{R}$.

Examples ① Let $f(x) = 2x$, $f: \mathbb{R} \to \mathbb{R}$.

Then f is linear.

Let $a_1, a_2 \in \mathbb{R}$. Then $f(a_1 + a_2) = 2(a_1 + a_2) = 2a_1 + 2a_2$

$= f(a_1) + f(a_2)$.

Let $a, c \in \mathbb{R}$. $f(ac) = 2(ac) = c(2a) = cf(a)$.

So f is linear.