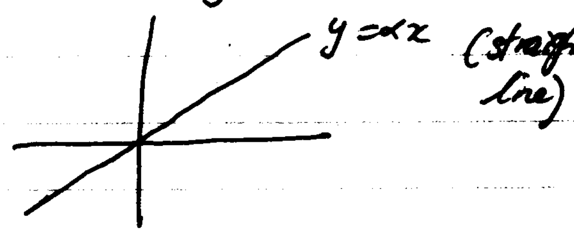
② Let $\alpha \in \mathbb{R}$ be a constant. Then the function $f(x) = \alpha x$, $f: \mathbb{R} \to \mathbb{R}$ is linear.



③ $L: \mathbb{R}^2 \to \mathbb{R}^2$ by $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ x_1 - x_2 \end{pmatrix}$ is a linear transformation.

Let $A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$. Then $L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

and $L(\vec{v}) = A\vec{v}$ where $A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$.

Let $\vec{v_1}, \vec{v_2} \in \mathbb{R}^2$ & $c \in \mathbb{R}$.

$$L(\vec{v_1} + \vec{v_2}) = A(\vec{v_1} + \vec{v_2}) = A\vec{v_1} + A\vec{v_2}$$
$$= L(\vec{v_1}) + L(\vec{v_2})$$
$$L(c\vec{v_1}) = A(c\vec{v}) = cA\vec{v} = cL(\vec{v_1})$$

and L is a linear transformation.

④ Let F = Set of differentiable functions $f(x)$

Let G = set of functions

Let $L: F \to G$ by $L(f) = f'$, where $f'(x) = \frac{d}{dx} f(x)$. Then L is linear

Let $f_1, f_2 \in F$ and $c \in \mathbb{R}$.

$$L(f_1 + f_2) = (f_1 + f_2)' = f_1' + f_2'$$
$$= L(f_1) + L(f_2)$$
$$L(cf_1) = (cf_1)' = cf_1' = cL(f_1)$$

& L is linear.