$L(f_1) + L(f_2) = x + 0 = 0.$

So $L(f_1 + f_2) \neq L(f_1) + L(f_2)$, &

L is not linear.

<u>Definition</u>: A linear 2^{nd} order DE has the form

$$(*) \quad a_2(x)\, y'' + a_1(x)\, y' + a_0(x)\, y = b(x).$$

<u>Example</u>

$$1 \cdot y'' + (\sin x) y' + (x^2+1) y = e^x$$

is a second order linear DE. This DE can be written as

$$L(y) = e^x$$

where $L(y) = y'' + (\sin x) y' + (x^2+1) y$, and L is linear transformation.

In general $(*)$ can be written as

$$L(y) = b(x)$$

where $L(y) = a_2(x) y'' + a_1(x) y' + a_0(x) y$ & L is a linear transformation.

<u>Theorem</u>: Suppose y_1, y_2 are solutions of the 2^{nd} order linear homogeneous DE

$$(**) \quad a_2(x)\, y'' + a_1(x)\, y' + a_0(x)\, y = 0.$$

Then $y = c_1 y_1 + c_2 y_2$ is also a solution if c_1, c_2 are constants.

<u>PROOF</u>: Let $L(y) = a_2(x) y'' + a_1(x) y' + a_0(x) y$.

Then L is a linear transformation.

We are given

$$a_2(x)\, y_1'' + a_1(x)\, y_1' + a_0(x)\, y_1 = 0$$

and $a_2(x) y_2'' + a_1(x) y_2' + a_0(x) y_2 = 0.$