Then $L(y_1) = 0$ and $L(y_2) = 0$.
Let $y = c_1 y_1 + c_2 y_2$ where c_1, c_2 are constants.
Then

$$L(y) = L(c_1 y_1 + c_2 y_2)$$
$$= L(c_1 y_1) + L(c_2 y_2) \quad \text{(since } L \text{ is linear)}$$
$$= c_1 L(y_1) + c_2 L(y_2) \quad \text{(since } L \text{ is linear)}$$
$$= c_1 0 + c_2 0$$
$$= 0,$$

ie $L(y) = 0$ and $y = c_1 y_1 + c_2 y_2$ is a solution to (**).

<u>Example</u>

(1) Show that $y_1 = \sin x$, $y_2 = \cos x$ are solutions to $y'' + y = 0$.

$y_1 = \sin x$ $\qquad$ $y_2 = \cos x$
$y_1' = \cos x$ $\qquad$ $y_2' = -\sin x$
$y_1'' = -\sin x$ $\qquad$ $y_2'' = -\cos x$.

$y_1'' + y_1 = \sin x - \sin x = 0$
$y_2'' + y_2 = \cos x - \cos x = 0$.
So $y_1 = \sin x$ & $y_2 = \cos x$ are solutions to the DE.

(2) What does the theorem imply?
The Theorem implies that
$$y = c_1 \sin x + c_2 \cos x$$
is also a solution to the DE if c_1, c_2 are constants.