EX (1) (i) Show that $y_1 = e^x$ and $y_2 = e^{-x}$ are solutions of the DE

$$y'' - y = 0.$$

(ii) What does the theorem imply?

(2) (i) Show that

$y_1 = \sin(\ln(x))$ & $y_2 = \cos(\ln(x))$ are solutions of the DE

$$x^2 y'' + xy' + y = 0 \qquad (x > 0).$$

(ii) What does the Theorem imply?

EXISTENCE UNIQUENESS THEOREM 1

Let $p(x)$, $q(x)$ be continuous functions on (a,b). Let x_0, y_0 be constants & Then $a < x_0 < b$

IVP $\quad y' + p(x)y = q(x), \quad y(x_0) = y_0$

has a unique solution on the interval (a,b).

EXISTENCE UNIQUENESS THEOREM 2

Let $p_1(x)$, $p_0(x)$, $q(x)$ be continuous functions on an open interval (a,b). Let x_0, y_0, y_1 be constants, Then where $a < x_0 < b$

IVP $\quad y'' + p_1(x)y' + p_0(x)y = q(x), \quad y(x_0) = y_0, \quad y'(x_0) = y_1$

has a unique solution on the interval (a,b).