Example

Solve the IVP

$$y'' + y = 0, \quad \cancel{y(0)=3}\ y(\pi/2) = 2, \quad y'(\pi/2) = -3.$$

We know

$$y = c_1 \sin x + c_2 \cos x \quad \text{is a soln of}$$

the DE for any constant c_1, c_2.

We want $y(\pi/2) = c_1 \sin \pi/2 + c_2 \cos \pi/2 = c_1 = 2.$

$$y' = c_1 \cos x - c_2 \sin x.$$

We want $y'(\pi/2) = -c_2 = -3$ & $c_2 = 3.$

So the solution is

$$y = 2 \sin x + 3 \cos x.$$

Note: This the only solution since it is unique & the solution is defined for all x.

Existence Uniqueness Theorem (3rd linear DE)

Let $p_2(x), p_1(x), p_0(x), q(x)$ be continuous functions on an open interval (a, b). Let x_0, y_0, y_1, y_2 be constants, where $a < x_0 < b$. Then the IVP

$$\begin{cases} y^{(3)} + p_2(x) y'' + p_1(x) y' + p_0(x) y = q(x), \\ y(x_0) = y_0, \quad y'(x_0) = y_1, \quad y''(x_0) = y_2 \end{cases}$$

has a unique solution on the interval (a, b).