## The General Existence Uniqueness Theorem for Linear DEs

Let $n \geq 1$ be an integer.
Let $p_0(x), p_1(x), \ldots, p_{n-1}(x), g(x)$ be continuous functions on an open interval $(a, b)$.
Let $x_0, y_0, y_1, \ldots, y_{n-1}$ be constants, where $a < x_0 < b$.
Then the IVP

$$\begin{cases} y^{(n)} + p_{n-1}(x)\, y^{(n-1)}(x) + \cdots + p_1(x)\, y'(x) + p_0(x)\, y(x) = g(x) \\ y(x_0) = y_0, \quad y'(x_0) = y_1, \ldots, \quad y^{(n-1)}(x_0) = y_{n-1} \end{cases}$$

has a unique solution on the interval $(a, b)$.

**Example** (Qu 2, Test 2, Spring 2001)
Use the Existence Uniqueness Theorem for $2^{nd}$ order linear DEs to discuss the existence & uniqueness of a solution to the IVP

$$x^2 \frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - 3y = \tan x, \qquad y(1) = 0, \; y'(1) = a/2.$$

$$DE \Leftrightarrow \quad \frac{d^2y}{dx^2} + \frac{(1-x)}{x}\frac{dy}{dx} - \frac{3}{x} = \frac{\tan x}{x} \qquad \text{for } x \neq 0.$$

Let $p_1(x) = \dfrac{1-x}{x}$ is continuous for $x \neq 0$.

$p_2(x) = -\dfrac{3}{x}$ is continuous for $x \neq 0$

$g(x) = \dfrac{\tan x}{x}$ is continuous for $x \neq 0, \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$