$p_1(x)$, $p_2(x)$ and $g(x)$ are all continuous on the interval $(0, \pi/2)$ which contains $x_0 = 1$.
The Theorem implies the IVP has a unique solution on the interval $(0, \pi/2)$ ie defined for $0 < x < \pi/2$.

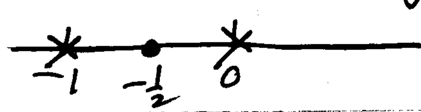
<u>Example</u> (#4, 6.1, p.324)
Determine the largest interval $(a,b)$ for which Theorem 1 (book) guarantees the existence of a unique solution on $(a,b)$ to the IVP

$$\begin{cases} x(x+1)\,y''' - 3xy' + y = 0, \\ y\left(-\frac{1}{2}\right) = 1, \quad y'\left(-\frac{1}{2}\right) = y''\left(-\frac{1}{2}\right) = 0. \end{cases}$$

The DE $\Leftrightarrow$ $y''' + 0y'' - \dfrac{3x}{x(x+1)}y' + \dfrac{1}{x(x+1)}y = 0$ for $x \neq 0, -1$.

$p_2(x) = 0$ is continuous for all $x$.
$p_1(x) = \dfrac{-3x}{x(x+1)}$ is continuous for $x \neq 0, -1$

$p_0(x) = \dfrac{1}{x(x+1)}$ is CTS for $x \neq 0, -1$
$g(x) = 0$ is CTS for all $x$.
all four functions are CTS for $x \neq 0, -1$.



All four functions are CTS on the interval $(-1, 0)$ and this (is the) largest interval that contains $x_0 = -\frac{1}{2}$.
Theorem implies the existence of a unique solution on the interval $(-1, 0)$.