Example

Determine if
$f_1(x) = \sin^2 x$, $f_2(x) = \cos^2 x$, $f_3(x) = 1$
are linearly independent on $(-\infty, \infty)$.
Observe that $\sin^2 x + \cos^2 x = 1$

$(1)\sin^2 x + (1)(\cos^2 x + (-1)\,1 = 0$

$1 f_1(x) + 1 f_2(x) + (-1) f_3(x) = 0$ for all $x \in (-\infty, \infty)$.

The functions are linearly dependent on $(-\infty, \infty)$.

Theorem

$f_1(x), f_2(x), \ldots, f_n(x)$ are linearly dependent on an interval I if some $f_j(x)$ can be written as a linear combination of the others; i.e.

$$f_j(x) = \sum_{i \neq j} d_i f_i(x) \qquad (\text{for } x \in I)$$

where the d_i are constants.

Example

$f_1(x) = 2x + 3$, $f_2(x) = x$, $f_3(x) = 1$
are linearly dependent since

$$f_1 = 2 f_2 + 3 f_3$$

f_1 is a linear combination of f_2 & f_3.
In fact,

$1 f_1(x) + (-2) f_2(x) + (-3) f_3(x) = 0$ for all x.