Example

$f_1(x) = 2x+3$, $f_2(x) = x$, $f_3(x) = 1$, $f_4(x) = e^x$ are linearly dependent on $(-\infty, \infty)$ since

$f_1(x) = 2f_2(x) + 3f_3(x) + 0\, f_4(x)$

for all x.

Prop. Two functions $f_1(x)$, $f_2(x)$ are linearly dependent on I iff (if and only if) one function is a constant times the other.

Example $f_1(x) = \frac{x}{2} + \frac{1}{x}$, $f_2(x) = 2x+2$ are linearly dependent on $(-\infty, \infty)$ since $f_1 = \frac{1}{4} f_2$.

Theorem

Let $p_1(x)$, $p_0(x)$ be continuous functions on an open interval (a,b).

If $y_1(x)$, $y_2(x)$ are linearly independent solutions of

$$(*)\quad y'' + p_1(x)y' + p_0(x)y = 0 \qquad \text{(HOMOGENEOUS DE)}$$

on (a,b) then the general solution of $(*)$ is given by

$$y = c_1 y_1(x) + c_2 y_2(x)$$

where c_1, c_2 are any constants.