<u>General Theorem</u> Let $n \geq 1$.
Let $p_0(x), p_1(x), \ldots, p_{n-1}(x)$ be continuous functions on (a,b).
If $y_1(x), y_2(x), \ldots, y_n(x)$ are linearly independent solutions of

$$(*) \quad y^{(n)} + p_{n-1}(x)\, y^{(n-1)} + \cdots + p_1(x) y' + p_0(x) y = 0$$

on (a,b) then the general solution of $(*)$ is given by

$$y = c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x)$$

where $c_1, c_2, \ldots, c_n$ are any constants.

<u>NOTE</u>: In this case $\{y_1(x), y_2(x), \ldots, y_n(x)\}$ is called a <u>fundamental solution set</u>.

<u>Example</u>:
We know $y_1 = \sin x$, $y_2 = \cos x$ are solutions to

$$(A) \quad y'' + y = 0$$

y_1, y_2 are linearly independent on $(-\infty, \infty)$ since $\dfrac{y_1}{y_2} = \tan x$ is not constant on any interval.

$y_1 = \sin x$, $y_2 = \cos x$ are linearly independent solutions of (A) on $(-\infty, \infty)$. They form a fundamental set of solutions. Hence the general solution of (A) is

$$y = c_1 \sin x + c_2 \cos x$$

where c_1, c_2 are any constants.