REVIEW of SOLVING TWO EQUATIONS IN TWO UNKNOWNS

Let a, b, c, d, e, f be constants.

$$(*) \begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

$$\iff \begin{aligned} acx + bcy &= ce \\ acx + ady &= af \end{aligned}$$

so $(ad-bc)y = (af-ce)$

& $y = \dfrac{(af-ce)}{(ad-bc)}$

assuming $ad-bc \neq 0$.

$$(*) \iff \begin{cases} adx + bdy = de \\ bcx + bdy = bf \end{cases}$$

so $(ad-bc)x = de - bf$

& $x = \dfrac{de-bf}{ad-bc}$ if $ad-bc \neq 0$.

DETERMINANT: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad-bc.$

Theorem Let a, b, c, d, e, f be constants.

If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$ then the system of equations

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

has the unique solution

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$