WRONSKIAN

The <u>Wronskian</u> of $y_1(x), y_2(x)$ is defined by

$$W[y_1, y_2] = W[y_1, y_2](x) := \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}.$$

<u>Example</u> Find the Wronskian of $y_1 = e^x$, $y_2 = e^{2x}$.

$$W[y_1, y_2] = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = 2e^{3x} - e^{3x} = e^{3x}.$$

<u>THEOREM</u> Suppose $y_1(x), y_2(x)$ are d'ble functions on an interval (a,b) & $a < x_0 < b$.
If $W[y_1, y_2](x_0) \neq 0$ then

$y_1(x), y_2(x)$ are linearly independent on (a,b).

<u>Proof</u> Suppose $W[y_1, y_2](x_0) \neq 0$.
Suppose c_1, c_2 are constants &

$$c_1 y_1(x) + c_2 y_2(x) = 0 \quad \text{for all } x \in (a,b).$$

Then $c_1 y_1'(x) + c_2 y_2'(x) = 0$.
Hence

$$\begin{aligned} y_1(x_0)\, c_1 &+ y_2(x_0)\, c_2 = 0 \\ y_1'(x_0)\, c_1 &+ y_2'(x_0)\, c_2 = 0 \end{aligned}$$

$c_1 = c_2 = 0$ is one solution. It is the ONLY solution ~~since~~ (ie unique) since $\begin{vmatrix} y_1(x_0) & y_2(x_0) \\ y_1'(x_0) & y_2'(x_0) \end{vmatrix} = W[y_1, y_2](x_0) \neq 0$.

Hence $y_1(x), y_2(x)$ are linearly independent on (a,b).