COROLLARY Let $y_1(x), y_2(x)$ be d'ble on (a,b).
If $y_1(x), y_2(x)$ are linearly dependent on (a,b) then $W[y_1, y_2](x) = 0$ for all $x \in (a,b)$

Example $y_1 = e^{x}$, $y_2 = 2e^{x}$
$y_1' = e^{x}$ $y_2' = 2e^{x}$.
y_1, y_2 are linearly dependent on $(-\infty, \infty)$ since $y_2 = 2y_1$.

$$W[y_1, y_2] = \begin{vmatrix} e^{x} & 2e^{x} \\ e^{x} & 2e^{x} \end{vmatrix} = 2e^{2x} - 2e^{2x} = 0 \text{ for all } x.$$

QUESTION: Is the converse of the corollary true?
NO
For example, $y_1(x) = x^3$, $y_2(x) = \begin{cases} x^3 & x \geq 0 \\ -x^3 & x < 0 \end{cases}$

are d'ble on $(-\infty, \infty)$, $y_1(x)$ & $y_2(x)$ are linearly independent on $(-\infty, \infty)$ BUT
$W[y_1, y_2](x) = 0$ for all x. (EX).

The Wronskian of $y_1(x), y_2(x), \dots, y_n(x)$ is defined by

$$W[y_1, y_2, \dots, y_n] = W[y_1, \dots, y_n](x) := \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ y_1'' & y_2'' & \cdots & y_n'' \\ \vdots & & & \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$