<u>Example</u> Find the Wronskian of
$y_1 = e^x$, $y_2 = \sin x$, $y_3 = \cos x$.

$$W[y_1, y_2, y_3] = \begin{vmatrix} e^x & \sin x & \cos x \\ e^x & \cos x & -\sin x \\ e^x & -\sin x & -\cos x \end{vmatrix}$$

$$= e^x \begin{vmatrix} \cos x & -\sin x \\ -\sin x & -\cos x \end{vmatrix} - \sin x \begin{vmatrix} e^x & -\sin x \\ e^x & -\cos x \end{vmatrix}$$

$$+ \cos x \begin{vmatrix} e^x & \cos x \\ e^x & -\sin x \end{vmatrix}$$

$$= e^x(-\cos^2 x - \sin^2 x) - \sin x(-\cos x\, e^x + \sin x\, e^x)$$
$$+ \cos x(-e^x \sin x - e^x \cos x)$$

$$= e^x(-1) - \sin^2 x\, e^x - \cos^2 x\, e^x = -2e^x$$

<u>Theorem</u>

Suppose $y_1(x), y_2(x), \dots, y_n(x)$ are $(n-1)$ times d'ble function on an open interval (a,b) & $a < x_0 < b$.
If $W[y_1, y_2, \dots, y_n](x_0) \neq 0$ then

$y_1(x), y_2(x), \dots, y_n(x)$ are linearly independent on (a,b).

<u>Corollary</u>. If $y_1, y_2, \dots, y_n$ are linearly dependent on (a,b) then
$W[y_1, y_2, \dots, y_n](x) = 0$ for all $x \in (a,b)$.