<u>Example</u> $y_1 = e^x$, $y_2 = \sin x$, $y_3 = \cos x$ are linearly independent on $(-\infty, \infty)$ since $W[y_1, y_2, y_3](x) = -2e^x \neq 0$ (for all x).

<u>Theorem</u>

Let $p_0(x), p_1(x), \ldots, p_{n-1}(x)$ be CTS on (a,b).
Suppose $y_1, y_2, \ldots, y_n$ are solutions of

$$(*)\quad y^{(n)} + p_{n-1}(x)\, y^{(n-1)} + \cdots + p_1(x)\, y' + p_0(x)\, y = 0$$

Then

<u>EITHER</u>

(1) $W[y_1, y_2, \ldots, y_n](x) \neq 0$ for all $x \in (a,b)$ & the solutions $y_1, y_2, \ldots, y_n$ are linearly independent on (a,b)

OR

(2) $W[y_1, \ldots, y_n](x) = 0$ for all $x \in (a,b)$ & the solutions $y_1, y_2, \ldots, y_n$ are linearly dependent on (a,b).

<u>NOTE</u>: In this situation

$$W[y_1, y_2, \ldots, y_n](x) = W[y_1, y_2, \ldots, y_n](x_0)\, \underbrace{e^{\int_{x_0}^{x} p_{n-1}(t)\, dt}}_{\neq 0}$$